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Spin-wave-mediated quantum corrections to the conductivity of thin ferromagnetic films of gadolinium

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We present a study of quantum corrections to the conductivity of thin ferromagnetic gadolinium films. *In situ* magnetotransport measurements were performed on a series of thin films with thickness $d<150$ Å. For sheet resistances $R_0 \leq 2840\ \Omega$ and temperatures $T \leq 30$ K, we observe a localizing, *linear* in temperature contribution to the conductivity in addition to the logarithmic temperature dependence expected from well-known quantum corrections in two dimensions. We show that such a linear T dependence can arise from a spin-wave-mediated Altshuler-Aronov-type correction.

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The observation of weak localization (WL) effects in percolating Ni films,[1] thin Fe films,[2] and ferromagnetic (GaMn)As nanostructures[3] is somewhat surprising since the magnetic fields associated with the magnetic domains might be expected to destroy the requisite quantum interference of self-intersecting electron trajectories. Since WL effects are cut off by various temperature-independent phase breaking scatterings and especially by the magnetic field inside the ferromagnet, one needs a sufficiently large temperature-dependent phase relaxation rate $1/\tau_\varphi$ to have an experimentally accessible disorder and temperature interval where such effects can be observed. Recent studies of the anomalous Hall (AH) effect in thin Fe films[2] have provided strong evidence for large temperature-dependent phase relaxation rates and hence observable WL effects in disordered ferromagnetic films. While the contribution to $1/\tau_\varphi$ in ferromagnetic films from electron-electron (e-e) interactions is small, a much larger contribution is obtained from scattering off spin waves,[4,5] such that the characteristic logarithmic temperature dependence of the conductivity due to WL effect was observed in polycrystalline Fe films within a range of temperature $5 \leq T \leq 20$ K, for sheet resistances measured at 5 K having values less than 3 kΩ.[2]

Given the importance of spin waves in Fe films, one expects to see an even larger effect in other ferromagnetic films with larger and more strongly coupled magnetic moments. In particular, quantum corrections to the conductivity due to the scattering off spin waves should be observable, just like the quantum corrections due to e-e Coulomb interactions, if the exchange coupling is large enough and the spin-wave gap is smaller than the temperature. It turns out that an excellent candidate is gadolinium (Gd), with a spin-wave gap of about 30 mK and a Curie temperature of 293 K.[6,7]

We have carried out systematic *in situ* magnetotransport measurements on a series of Gd films with varying thicknesses ($35<d<150$ Å) having sheet resistances ranging from 370 to 2840 Ω. We estimate that for these samples the phase coherence length $L_\varphi = \sqrt{D\tau_\varphi}$, where D is the diffusion coefficient, is larger than the sample thickness, so the films are effectively two dimensional. For temperatures $5 \leq T \leq 30$ K, we observe the simultaneous presence of two types of quantum corrections to the Drude conductivity: one has the expected logarithmic temperature dependence that is a hallmark of quantum corrections in two dimensions;[8] the other has a heretofore-unobserved approximately linear temperature dependence. We emphasize that the observed linear in T contribution is of a localizing nature and is therefore not caused by any of the usual known inelastic processes (phonons, e-e interactions). The only known origin of such a quantum correction besides WL is the Altshuler-Aronov-type correction. The latter, however, in its usual form based on Coulomb interaction between electrons is known to lead to a different temperature dependence, $\ln T$ in two dimensions. We argue that the situation is different in a ferromagnet, where the exchange of spin-wave excitations gives rise to an additional effective e-e interaction, leading to a heretofore unrecognized type of Altshuler-Aronov contribution. We calculate this spin-wave contribution within a standard diagrammatic perturbation theory and show that the results agree with the experimentally observed temperature and disorder dependence.

A series of ultrathin films of Gd in the Hall bar geometry was grown by rf magnetron sputtering through a shadow mask onto sapphire substrates held at a temperature of 240 K. The current and voltage leads of the deposited sample overlapped with predeposited palladium contacts, thus allowing reliable electrical connection for *in situ* measurements of the electrical properties. The experiments were performed in a specialized apparatus in which the sample can be transferred without exposure to air from the high-vacuum deposition chamber to a 7 T magnet located in a low-temperature cryostat. To parameterize the amount of disorder in a given film,[2] we use sheet resistances $R_0 \equiv R_{xx}(T=5\ \text{K})$, where R_{xx} is the longitudinal resistance. R_0 spans the range from 370 Ω (150 Å thick) to 2840 Ω ($\approx$35 Å thick). The use of sheet resistance to specify disorder in two-dimensional films[2,8–10] is an accepted practice based on theory and experiment. Longitudinal and Hall resistances were measured using standard four-probe lock-in techniques.

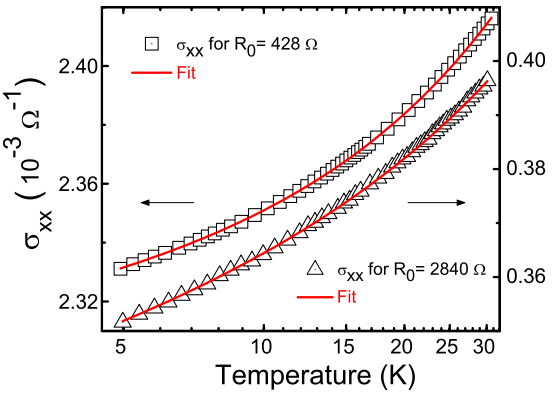

FIG. 1. (Color online) Temperature dependence on a logarithmic scale of σ_{xx} for two Gd thin films having sheet resistances R_0=428 Ω (open squares, left hand axis) and R_0=2840 Ω (open triangles, right hand axis). The solid-line fits for each curve are obtained using Eq. (1).

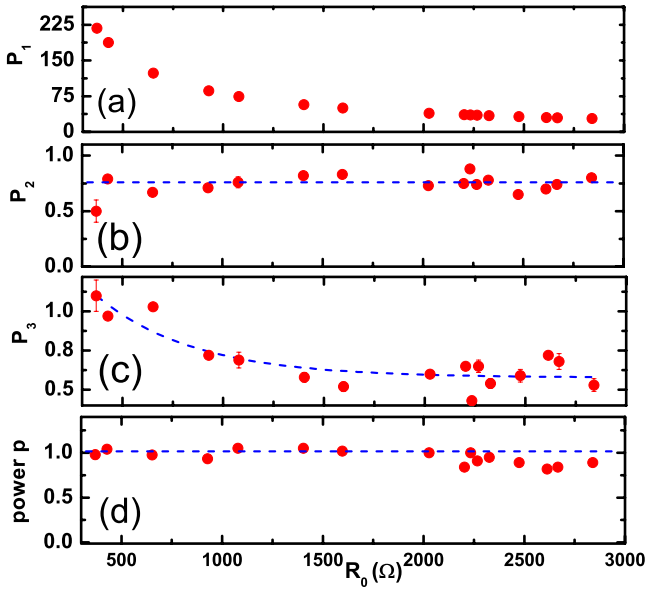

FIG. 2. (Color online) Dependence of the fitting parameters (P_1, P_2, P_3, p) defined in Eq. (1) on the disorder parameter R_0 for 16 thin-film samples. The horizontal dashed lines in panels (b) and (d) represent, respectively, the averaged values of P_2 and p, whereas the curved dashed line in panel (c) is a guide to the eyes.

Figure 1 shows the temperature dependence on a logarithmic scale of the longitudinal conductivity σ_{xx} for two typical thin Gd films with sheet resistances R_0 indicated on the figure. A fitting function of the form

$$\sigma_{xx}/L_{00} = P_1 + P_2 \ln(T/T_0) + P_3(T/T_0)^p, \tag{1}$$

where $L_{00}=e^2/\pi h$ and T_0=5 K is a reference temperature, has been used to fit each of the curves. The high quality of the fits shows that both logarithmic and linear temperature dependences are present; neither dependence alone describes the data.

The four fitting parameters, P_1, P_2, P_3, and p, for 16 different films with disorder parameters R_0 varying by almost a factor of 8 are plotted as a function of R_0 over the temperature range $5<T<30$ K in Fig. 2. In most cases the error bars are comparable to the symbol size. There are several important points about the fits that are worth emphasizing. First, the power p is close to unity over the indicated range where R_0 and, by implication, P_1 change by almost a factor of 8. (At higher disorder, $R_0>4$ kΩ, the power p decreases significantly.) Second, the coefficient P_3 decreases with increasing disorder by almost a factor of 2 and then saturates. Third, the prefactor P_2 of the logarithmic term, which is attributed to a combination of localization and interaction corrections (see below), is constant near unity over the displayed range. Finally, we have analyzed the same data set over the restricted temperature range $5<T<20$ K and found negligible change in the values of the fitting parameters and their respective errors. It does not make sense to include temperatures higher than 30 K since conductivity contributions from other mechanisms such as electron-phonon scattering become significant.

We will show that the unusual temperature dependence revealed by these data is consistent with a sum of contributions from well-known quantum corrections in two dimensions and a spin-wave-mediated correction analogous to the Altshuler-Aronov electron-electron contribution in disordered systems.[9] While the Altshuler-Aronov contribution gives rise to a logarithmic temperature dependence in two dimensions, we show that the spin-wave-mediated contribution can be linear in temperature within certain ranges of the parameters, consistent with the experiments. The theory ceases to be valid for large disorder ($R_0>4$ kΩ), where the temperature dependence is no longer linear.

To make comparisons with earlier studies on Fe films,[2,10] we have also measured the AH resistances of the Gd films at 7 T magnetic field. Following Ref. 10, we define normalized relative changes,

$$\Delta^N(Q_{ij}) \equiv (1/L_{00}R_0)(\delta Q_{ij}/Q_{ij}), \tag{2}$$

with respect to a reference temperature T_0=5 K$<T$, where $\delta Q_{ij}=Q_{ij}(T)-Q_{ij}(T_0)$ and Q_{ij} refers to either resistance R_{xx}, R_{xy} or conductivity σ_{xx}, σ_{xy}.

Figure 3 shows $\Delta^N(\sigma_{xy})$, together with $\Delta^N(R_{xx})$ and $\Delta^N(R_{xy})$ for comparison, for two different sheet resistances. We find that within the range of disorder, $\Delta^N\sigma_{xy}\approx 0$ for $5<T<20$ K. As shown in Ref. 11, the interaction correction to the AH conductivity is exactly zero due to symmetry reasons. This is true for both repulsive Coulomb interaction and the attractive spin-wave-mediated interaction. However, the WL correction to the AH conductivity need not be zero. In fact, the total WL contribution is given by

$$\Delta^N\sigma_{xy}^{\mathrm{WL}} = \frac{1}{1+r_{xy}}\ln\frac{T}{T_0}, \quad r_{xy} \equiv \frac{\sigma_{xy}^{sj}}{\sigma_{xy}^{ss}}, \tag{3}$$

where r_{xy} is the ratio of two different mechanisms contributing to the AH conductivity, namely, the side jump[12] σ_{xy}^{sj} and the skew scattering[13] σ_{xy}^{ss}. Thus the ratio r_{xy} is a nonuniversal quantity. It turns out that for Fe films, while $r_{xy}\ll 1$ for films deposited on glass, the opposite $r_{xy}\gg 1$ is true for films deposited on sapphire[2] or on antimony.[10] Our current results

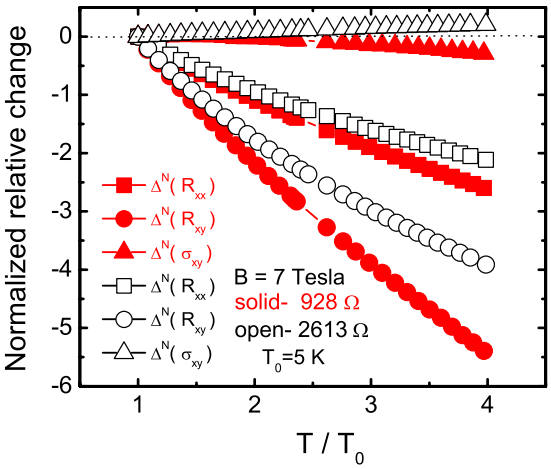

FIG. 3. (Color online) Normalized relative changes $\Delta^N(\sigma_{xy})$, defined in Eq. (2), for two different sheet resistances. For comparison we also show $\Delta^N(R_{xx})$ and $\Delta^N(R_{xy})$.

$\Delta^N\sigma_{xy}\approx 0$ for Gd deposited on sapphire agree with those of Fe films deposited on the same substrate. As for the longitudinal part, the coefficient A_R defined by

$$\Delta^N\sigma_{xx} = A_R \ln(T/T_0) \tag{4}$$

is given as $A_R=A_R^{\mathrm{WL}}+A_R^I=1+h_{xx}$, where the first term $(A_R^{\mathrm{WL}}=1)$ is due to WL and the second term $(A_R^I=h_{xx})$ is the exchange plus Hartree interaction contribution, with $h_{xx}=(1-3\tilde{F})/4$, where $\tilde{F}$ is the Hartree term. It has been argued that at least for Fe films, the total interaction correction h_{xx} is very small due to a near cancellation of the exchange and Hartree terms, which is expected due to strong screening. This results in $A_R\approx 1$ for Fe films. Figure 2 shows that $P_2=A_R\approx 0.75$ for our Gd films. This suggests that h_{xx} may actually be negative due to an even larger Hartree contribution. Since both r_{xy} and h_{xx} are nonuniversal quantities, we only note that a large r_{xy} (similar to Fe films on sapphire) and a small negative h_{xx} (large Hartree term, again similar to Fe films) are consistent with the current experimental observations (on Gd films on sapphire).

To understand the linear T dependence of the longitudinal conductivity, we evaluate the spin-wave contributions within the standard diagrammatic perturbation theory. The film is described as a quasi-two-dimensional system of conduction electrons with Fermi energies $\epsilon_{F\sigma}$ depending on the spin index $\sigma=\uparrow,\downarrow$. We model the total impurity potential as a sum over identical single impurity potentials $V(\mathbf{r}-\mathbf{R}_j)$ at random positions $\mathbf{R}_j$. The Hamiltonian is given by

$$\begin{aligned} H = &\sum_{\mathbf{k}\sigma}\left(\epsilon_{\mathbf{k}}-\frac{1}{2}\sigma B\right)c^+_{\mathbf{k}\sigma}c_{\mathbf{k}\sigma} \\ &+\sum_{\mathbf{k},\mathbf{k}'\sigma,\mathbf{j}} V(\mathbf{k}-\mathbf{k}')e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{R}_j}c^+_{\mathbf{k}'\sigma}c_{\mathbf{k}\sigma}+\sum_q \omega_q a^+_q a_q \\ &+J\sum_{q,k}[a^+_q c^+_{k+q\downarrow}c_{k\uparrow}+\mathrm{H.c.}] \end{aligned} \tag{5}$$

where c_k and c_k^+ are electron field operators, a_q and a_q^+ are spin-wave operators, and J is the effective spin-exchange

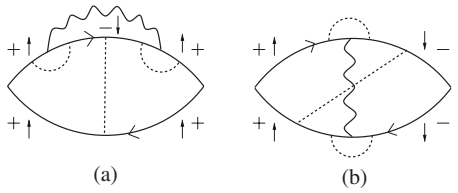

FIG. 4. Spin-wave contributions to the longitudinal conductivity. Solid lines are impurity averaged Green's functions, broken lines are "diffusons," and wavy line represents the effective spin-wave-mediated interactions. There are two diagrams of type (a) and four diagrams of type (b)

interaction. The spin wave is characterized by $\omega_q=\Delta_g+Aq^2$, where $\Delta_g\approx\mu_B B_{\mathrm{ext}}\approx 1$ K/T is the spin-wave gap and $A\approx J/k_F^2$ is the spin stiffness. Later we will drop Δ_g as $\Delta_g<T$. The exchange splitting $B\approx Jk_F^2$ is large, but $B/\epsilon_F\ll 1$, where we have used the values $B=700$ meV at 20 K (Ref. 14) and $\epsilon_F=3.4$ eV.[7] The spin-wave propagator is

$$S^{\mathrm{SW}}_{\uparrow\downarrow}(q,\omega_n) = 1/(i\omega_n - a\omega_q) = [S^{\mathrm{SW}}_{\downarrow\uparrow}]^*, \tag{6}$$

where $\omega_n=2\pi nT$ is the bosonic Matsubara frequency, $a=1-i\gamma/2$, and γ is a phenomenological damping constant. The spin-wave-mediated effective interaction is given by $V_{\mathrm{SW}}(q,\omega_n)=nJ^2[S^{\mathrm{SW}}_{\uparrow\downarrow}(q,\omega_n)+S^{\mathrm{SW}}_{\downarrow\uparrow}(q,\omega_n)]$ which is attractive. Here n is the density of conduction electrons.

For the quantum corrections to the conductivity, the dominant contributions from spin-wave interactions come from the diagrams with the most number of diffusons, analogous to those relevant for the Coulomb interactions. We therefore first evaluate the diffuson propagator $\Gamma^{\uparrow\downarrow}(q,\omega)$ in the presence of a large exchange splitting and obtain

$$\Gamma^{\uparrow\downarrow}(q,\omega_n) = 1/[(2\pi N_0\tau\hat{\tau})(\omega_n - iB + \hat{D}q^2)], \tag{7}$$

where $N_0=m/2\pi$ is the density of states at the Fermi surface, with m being the electron mass, τ is the scattering time, and we have defined

$$\hat{D}\equiv D(\hat{\tau}/\tau)^2, \quad 1/\hat{\tau}\equiv 1/\tau+\omega_n-iB, \tag{8}$$

where $D=(1/2)v_F^2\tau$ is the diffusion coefficient. The corresponding $\Gamma^{\downarrow\uparrow}(q,\omega_n)$ is obtained by replacing B by $-B$ everywhere.

From the diagrams shown in Fig. 4, the spin-wave contribution to the longitudinal conductivity is given, e.g., for Fig. 4(a), by

$$\begin{aligned} \delta\sigma^{\mathrm{SW}}_{xx\uparrow\downarrow} = T\sum_{\omega_n}\int_0^{q_c}\frac{dq^2}{(2\pi)^2}q^2v_F^2(2\pi N_0\tau\hat{\tau}^2)^2(2\pi N_0\tau)^2 \\ \times[\Gamma^{\uparrow\downarrow}(q,\omega_n)]^3 S^{\mathrm{SW}}_{\uparrow\downarrow}(q,\omega_n) \end{aligned} \tag{9}$$

and similarly for the $\downarrow\uparrow$ case. Here the upper cutoff in the q integral is given by $q_c=|1/v_F\hat{\tau}|$. The total spin-wave contribution can then be written as

$$\frac{\delta\sigma^{\mathrm{SW}}_{xx}}{L_{00}} \approx \frac{4N_0J}{1+\gamma^2/4}\frac{nJ}{B}\frac{\epsilon_F}{B}(\epsilon_F\tau)\frac{T}{Ak_F^2}, \tag{10}$$

where we have assumed that $B\tau\gg 1$.

Using $n=k_F^2/4\pi$, we estimate $\delta\sigma_{xx}/L_{00}\approx(\frac{Jk_F^2}{2\pi B})^2(\epsilon_F\tau)\frac{T}{Ak_F^2}$ for small damping. With an estimate of $\epsilon_F\tau\sim 10$ and $(Jk_F^2)/(2\pi B)\sim 1$, the observed magnitude of the constant $P_3\approx 1$ is consistent if $T_0/Ak_F^2\sim 1/10$, which is quite reasonable. On the other hand, the disorder dependence of the linear T contribution is given by $P_3\propto\epsilon_F\tau$, which decreases with increasing disorder. As observed experimentally in panel (c) of Fig. 2, P_3 does indeed decrease weakly with disorder up to a sheet resistance $R_0\approx 2000\ \Omega$ and then appears to saturate at a fixed value. We note that while the linear T behavior is observed to be quite robust for weak disorder, it cannot explain the data for $R_0>4\ \mathrm{k}\Omega$ (not shown in this Rapid Communication). We expect that for higher sheet resistances the system will undergo an Anderson localization transition from the pseudometallic phase (localization length longer than the sample dimensions) to a truly localized phase. The study of that regime is the subject of a forthcoming publication.

In conclusion, we have studied charge transport in ultrathin films of Gd grown using *in situ* techniques which exclude in particular unwanted oxidation or contamination. In addition to the logarithmic temperature dependence expected from weak localization effects in the longitudinal conductivity as previously seen in Fe films, we observe an additional contribution to the conductivity that has an approximately linear T dependence for sheet resistances $370\leq R_0\leq 2840\ \Omega$ and temperatures $5\leq T\leq 30$ K. We interpret this feature in terms of contributions from scattering off spin waves that are known to be important in ferromagnetic films. We find from our calculations that the interaction of the electrons with spin waves of the ordered ferromagnet gives rise to a much larger contribution than the usual one generated by the Coulomb interaction. The temperature dependence is governed by the singular (in the limit $\omega,q\rightarrow 0$) spin-wave propagator. The dressing by diffuson lines changes the prefactor but leaves the temperature dependence unchanged. To our knowledge, this type of quantum correction has not been seen and explained.

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